The market efficiency in the stock markets

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Abstract. We study the temporal evolution of the market efficiency in the stock markets using the complexity, entropy density, standard deviation, autocorrelation function, and probability distribution of the log return for Standard and Poor's 500 (S&P 500), Nikkei stock average index, and Korean composition stock price index (KOSPI). Based on a microscopic spin model, we also find that these statistical quantities in stock markets depend on the market efficiency.

1 Introduction

Econophysics is one of the most active fields in interdisciplinary research [1,2,3,4,5,6,7,8,9,10,11,12,13]. Time series analysis and agent based modelling have been studied by many researchers. There are many methodologies to analyze the financial time series. Observing probability distribution functions (FDFs) of log return is one of the simplest and the most popular methods. Many research papers about PDFs of log return for stock markets have already been published [10,11,12,13,14,15]. The dif-

ferent characteristics between mature markets and emerging markets [11], market efficiency [12], and the relation between shape of PDFs and time lags [13] are studied using PDFs. Also it is used to distinguish between bubble and anti-bubble [9,10].

Another method is computational mechanics [16]. Computational mechanics has been studied various fields of science [17,18,19], and it is applied to analyze the stock market [7]. Computational mechanics is available to analyze complexity and structure quantitatively by finding intrinsic causal structures of time series [20].

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Agent based modelling has been widely used in social science and econophysics to construct artificial social and economic systems. Agent based models in econophysics are constructed using agents clustering [1], Ising-like spin model [2,12], and Potts-like model [4]. Variation of PDFs shapes by traders' characteristics [10] and information flow [12], and speculative activity explaining bubbles and crashes in stock market [5] have been simulated by agent based model.

In this paper, we analyze the time series of Standard and Poor's (S&P 500), Nikkei stock average index, and Korean composition stock price index (KOSPI) by time evolution of statistical measures such as PDFs of log return, autocorrelation function, complexity, entropy density, and scaling properties of the standard deviation of log return. Moreover, we construct the stock market using microscopic spin model to simulate above time series results.

2 Empirical data and analysis

We use the S&P 500 data mainly for the period from 1983 to 2006. Japanese data for the period from 1997 to 2005 and Korean data for the period from 1992 to 2003 are also used to support and confirm the results from S&P 500. The data resolution is high frequency (1 minute) data, and we use only intra-day returns to exclude discontinuity jumps between the previous day's close and the next day's open price due to the overnight effects. The price return

is defined as

$$S(t) \equiv \log Y(t + \Delta t) - \log Y(t), \tag{1}$$

where $Y(t)$ is the price at time t and Δt is the time lag.

2.1 Probability distribution and autocorrelation

The distribution of price changes are identified as non-Gaussian [12,13,14,15,21]. Especially, when the PDF has the power law tail, the exponent of power at tail part can be gotten from the PDF. That exponent is called as tail index.
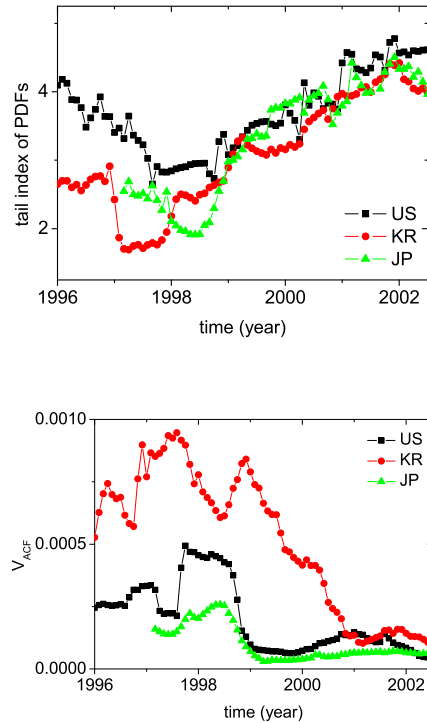

Fig. 1. (a) Temporal evolution of tail index and (b) variance of autocorrelation function for the S&P 500, the Nikkei stock index, and the KOSPI.

Fig. 1a shows temporal evolution of tail index in PDFs for S&P 500, Nikkei stock index, and KOSPI. Tail index of PDFs increases from around 2 to above 4 as time passes. In 2000s, the shape of PDF becomes narrower and the tail part becomes thinner, while PDF has fatter tail and the slope of tail part is more steep in 1990s. Autocorrelation function is defined as follows:

$$R(\tau) = \frac{< S(t)S(t+\tau) >}{\sigma^2}, \tag{2}$$

where σ is a standard deviation of $S(t)$. Moreover, the variance of autocorrelation function is defined as follows:

$$V_{ACF} = < R(\tau)^2 > . \tag{3}$$

Fig. 1b shows the temporal evolution of variance of autocorrelation function. The increasing tendency for tail index is reverse to it for variance of autocorrelation function. We can guess that the reason why probability distributions of log return are changed is related to autocorrelation of log return time series.

Though the tendency is same for three stock markets, the value of tail index for the S&P 500 is larger than it for the KOSPI and V_{ACF} for S&P 500 is smaller than it for the KOSPI in the 1990s.

2.2 Scaling property of standard deviation

We investigate the long range memory of log return by observing the time evolution of scaling properties in the standard deviation of log return [8]. The standard deviation of log return is defined as

$$\sigma(\Delta t) = \frac{\sqrt{\sum_{i=1}^{n} (\log Y(t_i + \Delta t) - \log Y(t_i))^2}}{\sqrt{n-1}}, \tag{4}$$
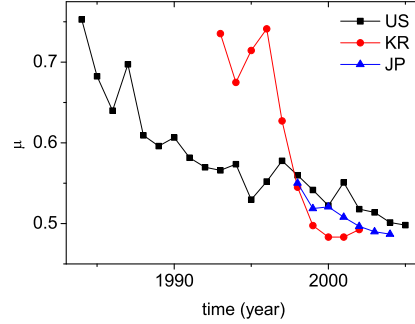

Fig. 2. Temporal evolution of scaling properties of the standard deviation of log return.

as a function of the time lag Δt. The relation between standard deviation and time lag is as follows:

$$\sigma(\Delta t) \sim \Delta t^{\mu}. \tag{5}$$

When μ is larger than 0.5, the time series has long range correlation, while long range anticorrelation when $\mu < 0.5$. There is no correlation at $\mu = 0.5$ and strength of correlation (or anticorrelation) is proportional to the difference between μ and 0.5. Fig. 2 shows the temporal evolution of scaling properties of the standard deviation of log return. The value of μ decreases to around 0.5. Until the mid 1990s, time series of stock market index has strong long range correlation. However, long range correlation practically disappears in 2000s.

In spite of the same tendency for temporal evolution of μ, the S&P 500 is more close to 0.5 than the KOSPI in the 1990s.

2.3 Entropy density and statistical complexity

We also analyze financial time series using computational mechanics to find the statistical complexity and the entropy density. In order to calculate statistical complexity, we used causal-state splitting reconstruction (CSSR) algorithm [16] to model ϵ-machine of the stock markets.

To calculate the entropy density and the statistical complexity, we should symbolize the time series as follows:

$$F(t) \equiv \theta(Y(t + \Delta t) - Y(t)), \qquad (6)$$

where $\theta(x)$ is a Heaviside step function. Then the original data $Y(t)$ are changed into the binary time series $F(t)$ with a countable set $A = \{0, 1\}$. $F(t)$ is 0 (or 1) when the next index has decreased (or increased).

Claude Shannon suggested the entropy of a discrete random variable X with a probability function $P(x)$ [22] as follows:

$$H[X] = -\sum_x P(x) \log_2 P(x). \qquad (7)$$

Let A be a countable set of symbols of time series and let S be a random variable for A, and s is its realization. If a block of string with L consecutive variable is denoted as $S^L = S_1, ..., S_L$, then Shannon entropy of length L is defined as

$$H[X] = -\sum_{s_1 \in A} \cdots \sum_{s_L \in A} P(s_1, ..., s_L) \log_2 P(s_1, ..., s_L). \qquad (8)$$

Also entropy density for the finite length L is define as

$$h_\mu(L) \equiv H(L) - H(L - 1), \qquad (9)$$

as a function of block length L where $L = 1, 2, 3, \cdots$. Entropy density is more useful because it is normalized quantity while $H(L)$ also increases as L increases.

In next, to calculate statistical complexity ϵ-machine has to be defined. An infinity string $\overleftrightarrow{S}$ can be divided into two semi-infinite parts such as a future $\overrightarrow{S}$ and a history $\overleftarrow{S}$. A causal state is defined as a set of histories that have the same conditional distribution for all the futures. ϵ is a function that maps each history to the sets of histories, each of which corresponds to a causal state:

$$\epsilon(\overleftarrow{s}) = \{\overleftarrow{s'} \mid P(\mathbf{S}^L = \mathbf{s}^L \mid \overleftarrow{S} = \overleftarrow{s}) = P(\mathbf{S}^L = \mathbf{s}^L \mid \overleftarrow{S} = \overleftarrow{s'}),$$
$$\mathbf{s}^L \in \mathbf{S}^L, \overleftarrow{s'} \in \overleftarrow{S}, L \in \mathbf{Z}^+\}. \qquad (10)$$

The transition probability $T_{ij}^{(a)}$ denotes the probability of generating a symbol a when making the transition from state S_i to state S_j [23,24].

The combination of the function ϵ from histories to causal states with the labelled transition probabilities $T_{ij}^{(a)}$ is called the ϵ-machine [23], which represents a computational model underlying the given time series.

Given the ϵ-machine, statistical complexity is defined as

$$C_\mu \equiv -\sum_{\{S_i\}} P(S_i) \log_2 P(S_i). \qquad (11)$$

Fig. 3 shows temporal evolution of statistical complexity and entropy density. Statistical complexity decreases and entropy density increases in all three markets as time passes.

Statistical complexity is around 0 when time series has regular pattern or it is totally random. To clarify whether the time series is random or regular, the entropy density is
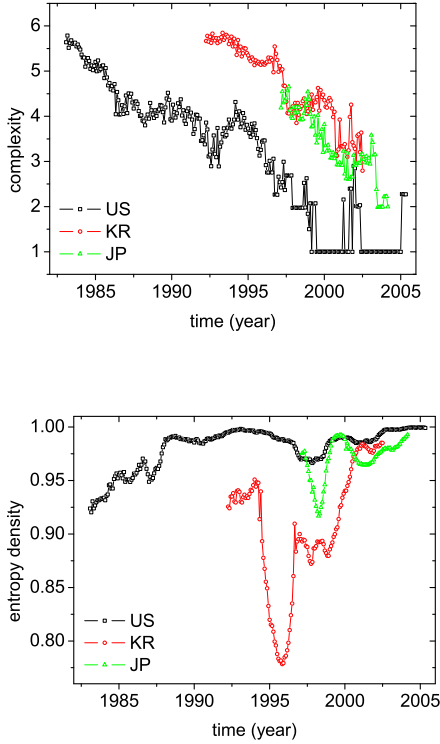
Fig. 3. Temporal evolution of (a) statistical complexity and (b) entropy density.

needed. Time series is totally random when entropy density is around 1 and entropy density is 0 when time series has periodic pattern because it is a measure of disorder. So we can find out that the time series of stock markets is getting more randomly and the patterns in the time series almost disappear in 2000s. Also the values of complexity and entropy density for the S&P 500 are different from them for the KOSPI, though the inclination is same.

3 Model and results

We constructed the microscopic model of many interacting agents to simulate the variation of some statistical characteristics for the stock price time series by modifying microscopic spin model [2]. The number of agents is N, and we consider $i = 1, 2, \ldots, N$ agents with orientations $\sigma_i(t) = \pm 1$, corresponding to the decision to buy $(+1)$ and sell (-1) stock at discrete time-steps t. The orientation of agent i at the next step, $\sigma_i(t + 1)$, depends on the local field:

$$I_i^{pri}(t) = \frac{1}{N} \sum_j A_{ij}(t)\sigma_j(t) + h_i(t), \qquad (12)$$

where $A_{ij}(t)$ represent the time-dependent interaction strength among agents, and $h_i(t)$ is an external field reflecting the effect of the environment. The time-dependent interaction strength among agents is $A_{ij}(t) = A\xi(t) + a\eta_{ij}(t)$ with $\xi(t)$ and $\eta_{ij}(t)$ determined randomly in every step. A is an average interaction strength and a is a deviation of the individual interaction strength. The external field reflecting the effect of the environment is $h_i = h\zeta_i(t)$, where h is an information diffusion factor, and $\zeta_i(t)$ is an event happening at time t and influencing the i-th agent.

From the local field determined as above, agent anticipates log return of stock index as follows:

$$x_i^{exp}(t) = \frac{2}{1 + e^{-2I_i^{pri}(t)}} - 1. \qquad (13)$$

So, the local field on agent can be represented as follows:

$$I_i(t) = I_i^{pri}(t) + \alpha \left(x(t-1) - x_i^{exp}(t-1) \right), \qquad (14)$$

where α is degree of adjustment. When $\alpha = 0$, agents determine their opinion from I_i^{pri}, while agents determine their opinion from the price or log return of previous step as well as information flowed into the market when α is non-zero. For instance, in case positive α and $x(t-1) > x_i^{exp}(t-1)$, agents determine their opinion by adding the

difference between the market price changes and the anticipated price changes at the previous step. On the contrary, in case $x(t-1) < x_i^{exp}(t-1)$, agents subtract the difference from I_i^{pri} to adjust their inexact information. By this way agents refer to past performance, while agents act by fundamental expressed by I_i^{pri} in case of $\alpha = 0$.

From the local field determined as above, agent opinions in the next step are determined by:

$$\sigma_i(t+1) = \begin{cases} +1 & \text{with probability } p \\ -1 & \text{with probability } 1-p \end{cases}, \quad (15)$$

where $p = 1/(1 + exp\{-2I_i(t)\})$. In this model, price changes are:

$$x(t) = \frac{1}{N} \sum \sigma_i(t). \quad (16)$$
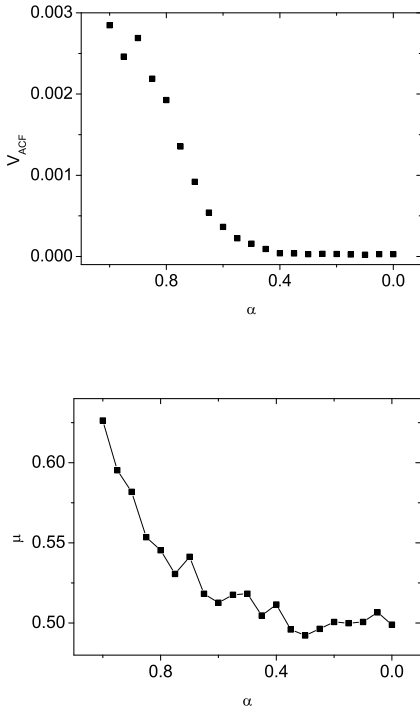

Fig. 4a shows variance of autocorrelation function for various α. As α decreases, the tail is getting thinner and thinner, and the strength of autocorrelation is reduced. Moreover, scaling exponents of standard deviation go to 0.5 as α decreases [see Fig. 4b]. The generated time series has long range correlation for larger α, and Correlation is almost disappeared for small value of α.

In Fig. 5, we can confirm the tendency of statistical complexity and entropy density for various α. As α decreases, statistical complexity decreases and entropy density increases. From the statistical complexity, the pattern in time series is getting simpler or the degree of randomness of times series is larger for smaller α. What entropy density is 1 means the time series is practically random. Fig. 5c is the relation between entropy density and statistical complexity. From this relation we can distinguish if the time series is random or regular.

4 Conclusions

We analyze the time series of stock index of U. S., Japan, and Korea using some statistical measures and simulate them by microscopic agent based spin model.

Time series has a fat tail in log return distribution and a tail index is increased as time passes to present. Existence of pattern in the financial time series can be confirmed by autocorrelation function, entropy density and complexity. As time goes from past to present, entropy density is increased and complexity is decreased. Also autocorrelation is decreased. From these results, the relation
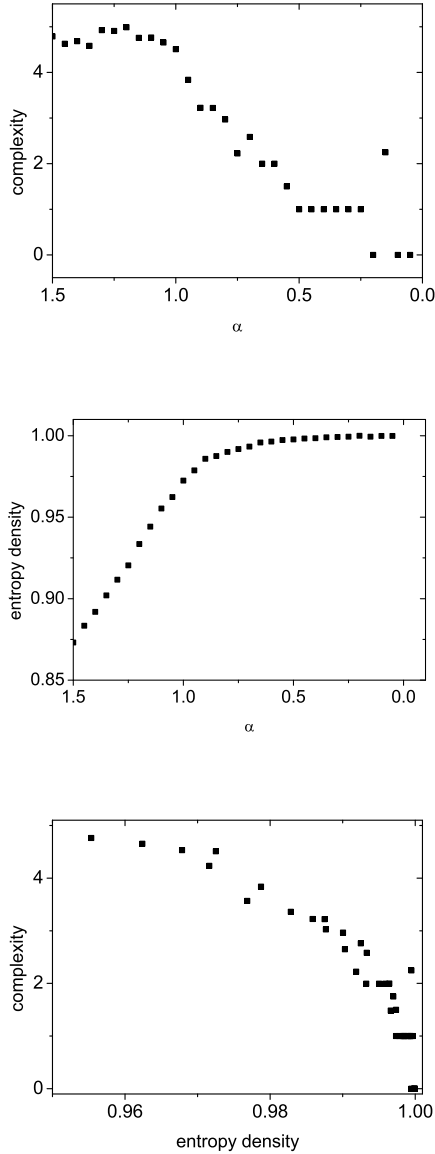
Fig. 4. (a) Variance of autocorrelation function for various and (b) Scaling exponents of standard deviation for various α.

Fig. 5. (a) Statistical complexity, (b) entropy density for various α, and (c) the relation between entropy density and statistical complexity.

between present data and past data is decreasing and the pattern in stock log return data disappears.

In the spin model, when α is non-zero, traders adjust their opinion using the difference between their anticipated prices and real market prices, and they antici-

pate price changes of next step with adjusted information. In the past, the speed of information is slower and market is less efficient, so adjusting behavior is more effective and active in the same time interval compare to present. Therefore, the past market corresponding to higher α has long range correlation and vice versa.

$I_i^{pri}(t)$ is generated randomly because its elements are random variables while $\alpha \left(x(t-1) - x_i^{exp}(t-1) \right)$ provides regularity to $I_i(t)$ because effect of this term remains for a while like Markov chain. When α is 0, entropy density is almost 1 and complexity is 0 because time series for $\alpha = 0$ are almost random. As α is increased, entropy density is decreased and complexity is increased because the pattern is generated in the time series.

The reason why these changes occur is that speed of information flow is becoming fast by the development of infra for communication such as high speed internet, mobile communication and broadcasting systems. So market has become more efficient. By the efficient market hypothesis (EMH), the speed of information is so fast that agents can not gain profit by superiority of information.

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